Simplified Sculptured-Surface Technique Applied to Wind-Tunnel Models

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I. Introduction

IN performing numerical-control (N/C) operations to generate a wind-tunnel model, it is essential that a mathematical description of the surface exists. The information necessary to N/C machine the surface is extracted from the mathematical model, and the degree of simplicity or complexity of the data extraction is directly related to the degree of simplicity or complexity of the mathematical model.

Many N/C machined sculptured surfaces are the result of laborious, expensive mathematical modeling programs. The technique described here is much less costly, and is somewhat similar to a low-cost program developed by Richard L. Simon.[1] Whereas Simon's approach did not develop an explicit mathematical expression for his surface, this program does develop such a description, which is then used to obtain coordinates and normal vectors for the surface. With this information, N/C machining on a 3- or 5-axis machine becomes possible with little additional labor. Some Fortran programming is required, however, the mathematical sophistication needed is minimal, and the operational cost is low.

Our surface definition permits simple computations of needed quantities and also facilitates handling changes in surface geometry. The basic definition procedure will be outlined. Our definition used two 3-dimensional space curves, one 2-dimensional space curve, and one modulating function.

II. Graphical Description

Before proceeding to a formal description of the surface development, a purely graphical construction will be outlined by the use of examples. The first surface to be described is a cylinder of constant cross-sectional area, with an axis parallel to the X-axis of the coordinate system (see Fig. 1). The cross section is described in successive planes perpendicular to the X-axis by the expression $H{:}Z^* = h(y^*)$. Z^* and y^* are the results of a scaling function, where y^* equals a sizing constant times y. A chord for the given surface can be described as the perpendicular distance between two space curves F and G. In Fig. 1, F and G are two lines parallel to the X-axis.

A second example is illustrated in Fig. 2. Here, the cross section of the shape is allowed to vary with X according to the equation $Z^* = h(y^*)\cdot \mathrm{d}(x)$. Note that F and G are parallel lines, as above, providing a surface of changing area and constant orientation in the coordinate space.

Our third example, illustrated in Fig. 3, shows that orientation of the surface can be changed by allowing F and/or G to vary with any coordinate in space. Here, G is as defined before, and F is a function of y and Z.

III. Mathematical Formulation of the Surface Definition Procedure

The two 3-dimensional space curves are depicted as follows:

$$F:\ \begin{cases} y = f1(x) \\ z = f2(x) \end{cases} \qquad G:\ \begin{cases} y = g1(x) & A \le x \le B \\ z = g2(x) \end{cases}$$

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The cross section is defined by the equation

$$z^* = h(y^*)\cdot \mathrm{d}(x), \qquad h(0) = h(1) = 0, \qquad \mathrm{d}(x) \ge 0$$

where

$$0 \le y^* \le 1 \qquad \text{and} \qquad A \le x \le B$$

The function $h(y^*)$ is selected to give the surface a desired shape character. The function $\mathrm{d}(x)$, which will be called the distribution function, is used to produce shape changes of the curve defined by $h(y^*)$ as x varies.

To develop the surface, the x-coordinate will be fixed, say for a value x, and a curve developed in this plane. Then the x-coordinate will be allowed to vary and the surface defined. A plane perpendicular to the x-axis at a value of x $(A \le x \le B)$ is considered. The curves F and G are intersected by this plane at the coordinate locations

$$[x, f1(x), f2(x)] \qquad \text{and} \qquad [x, g1(x), g2(x)],$$

respectively. Set:

$$a(x) = g1(x) - f1(x)$$

$$b(x) = g2(x) - f2(x)$$

$$c(x) = \sqrt{a(x)]^2 + [b(x)]^2}$$

and let θ be given by

$$\sin\theta = \frac{b(x)}{c(x)}, \quad \cos\theta = \frac{a(x)}{c(x)}$$

y^* and $z^* = h(y^*)\cdot \mathrm{d}(x)$ are scaled by the factor $c(x)$. In particular

$$x1 = x \qquad y1 = c(x)\cdot y^* \qquad z1 = c(x)\cdot \mathrm{d}(x)\cdot h(y^*)$$

This result is now rotated and translated, giving

$$x = x$$

$$y = f1(x) + \cos(\theta)\cdot y1 - \sin(\theta)\cdot z1$$

$$z = f2(x) + \sin(\theta)\cdot y1 + \cos(\theta)\cdot z1$$

or

$$x = x \tag{1a}$$

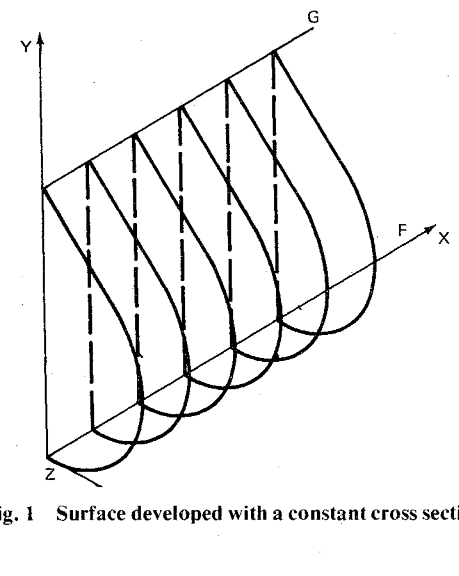

Fig. 1 Surface developed with a constant cross section.

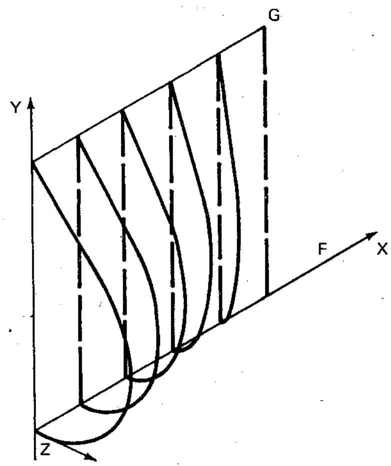

Fig. 2 Curves *F* and *G* are parallel and in the $x-y$ plane. d*(x)* varies with *x*.

$$y = f1(x) + a(x)$$

$$\cdot y^* - b(x) \cdot d(x) \cdot h(y^*) \quad (1b)$$

$$z = f2(x) + b(x) \cdot y^* + a(x) \cdot d(x) \cdot h(y^*) \quad (1c)$$

Letting x vary between A and B, the Eqs. (1) define the desired surface in terms of two parameters x and y^*.

If we introduce the radius vector $\boldsymbol{R}$ from the origin to a general point (x,y,z) of the surface, we may combine the three parametric Eqs. (1) into one vector equation,

$$\bar{R}(x,y^*) = x\bar{i} + y(x,y^*)\bar{j} + z(x,y^*)\bar{k}$$

where $A \leq x \leq B$ and $0 \leq y^* \leq 1$. At this point, the restriction that f, $f2$, $g1$, $g2$, h, and d be continuously differentiable is imposed. This insures continuity of $\partial\bar{R}/\partial x$ and $\partial\bar{R}/\partial y^*$ which implies a smooth surface from which normals will be obtained. In fact, the two vectors

$$\frac{\partial \bar{R}}{\partial x} = \bar{i} + \frac{\partial y}{\partial x}\bar{j} + \frac{\partial z}{\partial x}\bar{k} \quad \text{and} \quad \frac{\partial \bar{R}}{\partial y^*} = + \frac{\partial y}{\partial y^*}\bar{j} + \frac{\partial z}{\partial y^*}\bar{k}$$

determine the plane tangent to the surface. The cross product of these two vectors

$$\frac{\partial \bar{R}}{\partial x} x \frac{\partial \bar{R}}{\partial y^*} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & \dfrac{\partial y}{\partial x} & \dfrac{\partial z}{\partial x} \\ 0 & \dfrac{\partial y}{\partial y^*} & \dfrac{\partial z}{\partial y^*} \end{vmatrix} \quad (2)$$

is the desired normal vector. The possibility that the cross product is zero can be discounted since this can happen only if F and G intersect – a case which we will exclude.

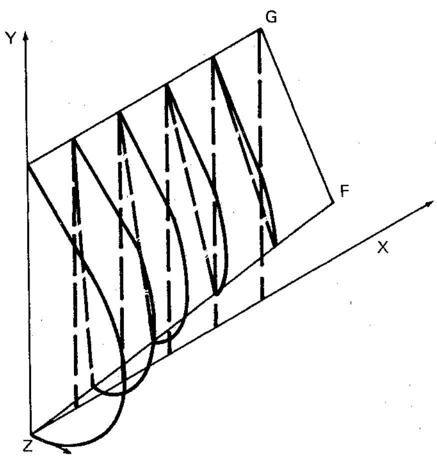

Fig. 3 *F* and *G* are not parallel. d*(x)* varies with *x*.

IV. Machining

The previous procedure was used recently to machine aerodynamic wind tunnel models at the Grumman Aerospace Corporation. One particular wing was defined by an airfoil section along with a distribution function and a leading and trailing edge with a built in twist distribution. This particular wing followed the outline for the surface development given above. Variations of this procedure will allow for a more complex geometrically defined wing.

The curves selected to define the leading and trailing edge, e.g., the curves F and G, were chosen to be conics, i.e., fi, gi, $i = 1,2$ are conic sections. This choice was made on the basis of available software and procedural capability in the handling of these curves. The function $d(x)$ was also represented by a sequence of conics, i.e., over a series of intervals $[a_1 a_2]$, $[a_2 a_3]$,. . ., $[a_{n-1}, a_n]$ different conics are used.

The airfoil was handled differently. When the program was developed, the exact defining curve procedure which the airfoil would take was uncertain. Because of this, and in an effort to avoid possible difficulties in dealing with a multiplicity of definition procedures, an input of a sequence of points with corresponding slopes was chosen for its simplicity. These points are sufficiently dense to provide a linear approximation to the airfoil shape. These points, in a fashion, will correspond to the output points via Eqs. (1). It is assumed that the input points are dense enough so that the output points give a good approximation of the surface along any plane given by $x = x_0 =$ constant.

This information, plus some obvious machining information such as cutter diameter, distance between cuts, feedrate, etc., is sufficient to produce a N/C machining tape to sculpture the surface. Tapes for the upper and lower surface are produced from the same program. The sole difference in the computational procedure is the offset direction. Since the program yields a definition for the entire wing, an upper or lower surface designation is in order.

Reference

[1] R. L. Simon, "A Sculptured Surface Program You Can Write Yourself," *Machine Design*, Aug. 1974, pp. 116-120.